Harmonic Distortion in Iron-Core Transformers

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A discussion of a simple method of measuring total harmonic distortion with accuracy adequate for routine check purposes.

One restriction upon the design of an audio transformer is that the inherent distortion be confined to insignificant proportions. It is therefore necessary to be able to predict the distortion introduced by a given transformer from data obtained from measurements upon samples of the core material. This information is particularly necessary in connection with large output transformers in which flux density may be quite high at the lowest frequency of the pass band.

Normally a transformer is driven from a circuit containing a thermionic valve and in considering the distortion introduced it is necessary to distinguish between (a) distortion introduced by the valve owing to its own non-linear characteristics and to the fact that it is working into a complex non-linear load; and (b) distortion introduced by the transformer owing to the non-linear characteristic of its core material. Only the latter type of distortion is here considered.

This particular problem has been investigated by N. Partridge[1] and it may be useful to re-state some of his conclusions:

(1) With respect to non-linear distortion in the transformer a circuit consisting of a source (internal resistance r), transformer and load, *Fig.* 1 may be replaced by the equivalent circuit of *Fig.* 2. In the case where all circuit elements are linear, this reduces to Thévenin's Theorem.

(2) If no constant polarizing current is present, only odd-number harmonics are produced. Third harmonic predominates and if an accuracy of 5 per cent is acceptable, higher harmonics may be neglected.

(3) The existence of a constant component of magnetizing force results in an asymmetrical hysteresis loop and even as well as odd harmonics appear. If the peak flux density is less than 10,000 gauss (for silicon-iron laminations) the harmonics above the third can be neglected with less than 5 per cent error.

(4) The percentage harmonic distortion appearing across a transformer in a circuit such as *Fig.* 2 is:

$$\frac{V_h}{V_f} = K\,S\frac{R_1}{f}\left(1-\frac{R_1}{Z_f}\right)$$

Where K is a numerical constant depending upon the core dimensions
S is a factor depending upon the core material and the peak flux density
R_1 is the value of Z_S and Z_L in parallel
Z_f is the primary open-circuit impedance at the fundamental frequency f.

Methods of Measuring Harmonic Distortion

Methods available for measuring distortion may be classified into three groups:

(1) Wave analyzer methods in which the amplitude (and perhaps the phase, too) of each frequency component in the distorted waveform is measured directly by a selective circuit. These methods give the fullest information but are slow and require expensive and bulky apparatus.

(2) Fundamental suppression methods in which the distorted voltage waveform is fed through a passive high-pass filter which rejects the fundamental but does not attenuate the harmonics. Such filters are not inexpensive and one filter is suitable for measurements at only one frequency.

(3) Fundamental suppression methods in which the distorted voltage waveform is balanced against a pure waveform at the fundamental frequency and the difference is measured. There are several methods of deriving the pure reference waveform by using special transformers or other passive networks, or by using vacuum tube circuits.

Simple Method for Measuring Total Distortion

A simple method for the rapid routine measurement of distortion has been developed by the authors and does not seem to be widely known. By the use of inexpensive apparatus total core distortion can be measured quickly under conditions strictly comparable with specified operating conditions.

The method falls into category (3) as listed above. *Figure* 3 shows a modification of (c) in *Fig.* 2 in which a parallel network composed of C_1, R, and r is connected to the same source of e.m.f. and a resistance R_2 is shown across the transformer primary inductance to represent the core losses.

With an indicator connected across A and B this will be recognized as Maxwell's Bridge. By adjustment of C_1, r, and R the fundamental voltage across AC can be made equal to that across BC. This setting will hold whatever the frequency of E so long as L and R are constant.

The balance conditions are then:

$$L = R\,R_1\,C_1$$

and:

$$R_Z = \frac{R\,R_1}{r}$$

In practice a perfect balance cannot be obtained since the non-sinusoidal magnetizing current through R_1 causes a distorted voltage waveform across R_1 (and L) and this cannot be completely balanced by the sinusoidal voltage across R. The residual voltage across AB when C_1, r, and R are adjusted for "balance" is the total harmonic distortion appearing across the transformer winding.

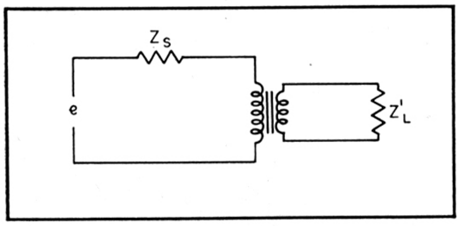

Fig. 1. Circuit of transformer working between impedances of Z_s and Z'_1.

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[1] N. Partridge, "Harmonic distortion in audio frequency transformers," *Wireless Engineer*, Sept. and Nov., 1942.

The measuring procedure then is as follows:

(1) Set R_1 to the value $Z_S Z_L/(Z_S + Z_L)$

(2) Set E to make the voltage across AC correspond to the desired power in the load Z_L; i.e., $(V_{AB})^2 = Z_L W_O$

(3) Adjust R and r to make the voltage across AB a minimum. This minimum reading is the harmonic voltage figure required, and if multiplied by $100/V_{AB}$ gives the percentage distortion factor. There is no need to read the values of R and r so that uncalibrated variable resistors may be used for these circuit elements.

Error of the Method

In the foregoing brief description it has been implicitly assumed that (1) the voltage source E is free of harmonics, (2) the voltage source has an internal impedance of zero, and (3) the "balance" indicator has an infinite input impedance and stray capacitances are negligible. Fortunately these conditions can be closely approached and in practice the errors introduced are of small order. Each source of error is examined separately below.

The conditions of balance for the Maxwell bridge contain no frequency

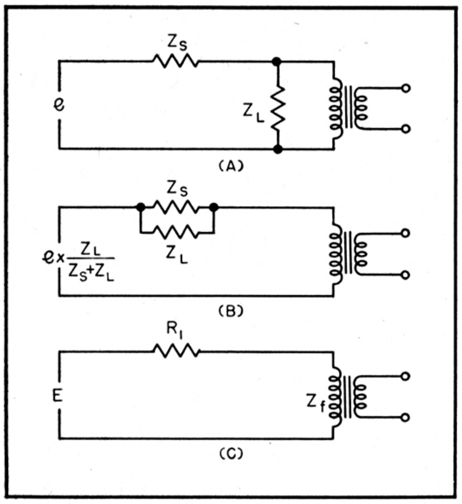

Fig. 2. Equivalent circuit of Fig. 1. (A) represents transposition of transformer impedance ratio; (B) represents transposition using Thévenin's theorem; and (C) represents simplification of (B).

term and so the bridge once balanced for one frequency remains so at all frequencies for which the numerical magnitude of the impedance elements remains unchanged. Unfortunately the inductance of an iron-cored transformer is considerably lower at the third harmonic than it is at fundamental frequency. Thus clearly can it be seen how harmonic frequency components of the source e.m.f. give a false value to the reading.

To examine the order of magnitude of this error let it be supposed that the bridge is balanced at the fundamental frequency and so the fundamental voltage across AB is zero. Now suppose that the inductance has a magnitude of L at fundamental frequency and $L\text{-}\delta L$ at third harmonic frequency. Let $\frac{n}{100} \times E$ be the magnitude of this third harmonic frequency present in the source (fundamental magnitude, E). The indicator across AB would give no reading of harmonic from the source if δL were zero. It will be seen therefore that the false reading due to $\frac{n}{100} E$ will be the difference between the fractions

$$\frac{j\omega(L\text{-}\delta L)}{R_1 + j\omega(L\text{-}\delta L)}$$

and $\frac{j\omega L}{R_1 + j\omega L}$ of the voltage, $\frac{n}{100}$ E;

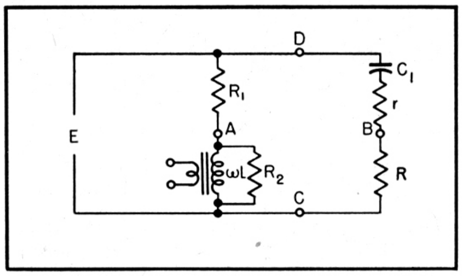

Fig. 3. Modification of (C) in Fig. 2 to permit of measurements of transformer characteristics.

that is

$$\left[\frac{j\omega(L\text{-}\delta L)}{R_1 + j\omega(L\text{-}\delta L} - \frac{j\omega L}{R_1 + j\omega L}\right]\frac{n}{100}E$$

Expressed as a percentage of the fundamental across the inductance, this is

$$\frac{\left[\frac{j\omega(L\text{-}\delta L)}{R_1 + j\omega(L\text{-}\delta L)} - \frac{j\omega L}{R_1 + j\omega L}\right]\frac{n}{100}E}{\frac{j\omega L}{R_1 + j\omega L}E}$$

$\times 100$ *per cent* which simplifies to

$$n \times \frac{\delta L}{L} \times \frac{1}{\sqrt{1 + \frac{\omega^2 (L\text{-}\delta L)^2}{R_1^2}}}$$

By means of this expression the false reading is related to the percentage harmonic, n, in the source and from a knowledge of the factors the value of n can be deduced for a given maximum permissible error reading. $\delta L/L$ is the fractional change of inductance as between fundamental and harmonic when the two are applied simultaneously. Under this condition it has been found experimentally that $\delta L/L$ is a small fraction of the order of a tenth to a fifth, measurements being taken with a fundamental frequency of 50 cps and third harmonic.

The order of magnitude of $1 + \frac{\omega^2 (L\text{-}\delta L)^2}{R_1^2}$ can be obtained from the knowledge that at any fundamental frequency in the pass band ωL will be numerically greater than $2R_1$ or at the third harmonic frequency greater than $6R_1$, which makes $\omega(L\text{-}\delta L)$ greater than $5R_1$; so that $\sqrt{1 + \frac{\omega^2 (L\text{-}\delta L)^2}{R_1^2}}$ becomes $\sqrt{26}$ which is near enough to 5.

The percentage n in the source gives rise to a false reading of magnitude less than 1/25th of n. So if a false reading of less than 0.1 per cent can be tolerated, the source must not contain more than 2½ per cent harmonic.

Error Due to Finite Source Impedance

Let it be supposed that the balanced bridge set-up driven from a pure tone source, of impedance R_S, is indicating an harmonic voltage across AB. The reading across AB can be considered as due to a fictitious generator of harmonics in series with L; as the source E does not affect the voltage across AB under balanced conditions it is short-circuited, giving the equivalent circuit of A in *Fig.* 4.

The true reading of distortion appears across AB when $R_S = 0$. How is this reading affected when R_S is not zero?

The complete analysis of this circuit leads into some rather tedious algebra but if the shunting effect of C and R across R_S can be ignored and if $L\omega > 2R_1$ when the equivalent circuit can be greatly simplified as follows:

(1) with respect to a voltage introduced as at e_n the voltage across AB is almost the same as the voltage across AD. This is easily seen by drawing the vector diagram of the various voltages.

(2) The arm CBD can therefore be completely omitted. A of *Fig.* 4 then reduces to e_n applied to L, R_1, and R_S in series. The indicated harmonic voltage is the voltage across R_1. In general, voltage across R_1 is $\frac{R_1}{(R_S + R_1) + j\omega L} \times e_n$

When $R_S = 0$ this becomes $\frac{R_1}{R_1 + j\omega L} \times e_n$, the true reading of harmonic voltage. The ratio of these two voltages is $\frac{R_1 + j\omega L}{R_S + R_1 + j\omega L}$. To test the magnitude of error introduced by R_S assume as before, $\omega L = 2R_1$ and also $R_S = 0.1R_1$. The fraction $\frac{R_1 + j\omega L}{R_S + R_1 + j\omega L}$ then reduces to $\frac{(1 + j2)}{(1.1 + j2)}$. The modulus of this ratio is $\frac{\sqrt{5}}{\sqrt{5.21}} = 0.98$, i.e., if R_S is 10 per cent of R_1 the error introduced into the reading is of the order of 2 per cent of the reading.

For many measurements the 60-cps line voltage provides a suitable source with an internal impedance of a few ohms.

Error Due to Stray Capacitances and Indicator Impedance

There is no difficulty in providing as an indicator a vacuum tube voltmeter of substantially infinite input impedance, and as distortion measurements are of importance only at low frequencies even large "strays" introduce negligible error. For example, 0.01 μf at 60 cps is an impedance of 0.27 megohms. However, the choice at grounding point on the bridge does need some consideration. The vacuum-tube voltmeter will probably have one input terminal which must be grounded and the best point for a ground on the bridge is A. The indicator can then be connected across AC, while the voltage V_{AC} is adjusted to give the appropriate flux density in the core, and then switched to AB while R and r are adjusted to give a minimum reading. The bridge must be driven from a floating winding on a transformer and the capacitance of this winding to ground will shunt one or more of the bridge elements.

A vacuum-tube voltmeter with three-terminal input being developed by the authors will give greater freedom in

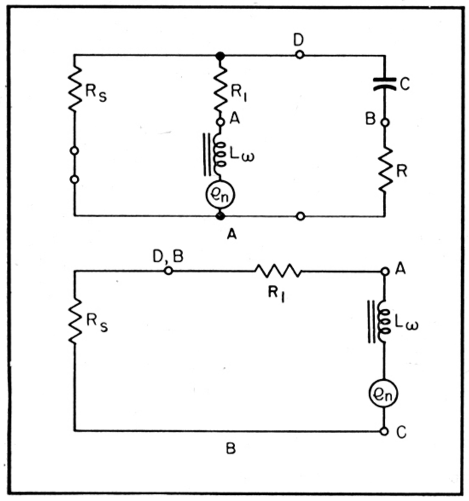

Fig. 4. Bridge of Fig. 3 driven by generator of finite source impedance. (A) actual circuit, and (B) equivalent circuit.

measurements of this kind. Suppose the three input terminals are T_A, T_B, T_C. The terminal T_C is permanently grounded and the input impedance between any two terminals is very high. The indicator reads the voltage between T_A and T_B irrespective of any voltage $T_A - T_C$ or $T_B - T_C$.

Choice of Components

Summing up, the method offers a means of rapid measurement of total distortion using a minimum number of inexpensive components and of sufficient accuracy for most purposes.

The resistance R_1 may be built up from selected composition resistors as the impedance it represents is not known to a high degree of accuracy. A fixed tubular paper capacitor of suitable value is used for C_1. The resistance R may conveniently be a chain of fixed value composition resistors in series with a composition potentiometer, giving a coarse adjustment and a fine adjustment over a wide range. Another composition potentiometer is suitable for r. For distortion measurements the elements C_1, r, and R need not be calibrated; they are merely adjusted so that the indicator reading is at a minimum.

Strictly speaking, the indicator itself should give an indication of the r.m.s. value of the complex waveform of distortion voltage appearing across AB. Usually the third harmonic predominates and in this case the mean value, as indicated on a rectifier-type moving-coil instrument, will be close to the r.m.s. value.

If laboratory type decade boxes can be used in the set-up, and a suitable source of variable frequency is available, it may be noted that all the other important parameters of transformer performance can be measured with only a small change of set-up.

(1) *Incremental primary inductance and core loss.*

Use as Maxwell bridge. Frequency as for distortion measurement but voltage reduced.

(2) *Primary leakage inductance and copper loss.*

Short circuit secondary of component on test and increase frequency to middle of pass-band. Use as Maxwell bridge. For this measurement it is more convenient to have C_1 and r in parallel giving the leakage inductance and equivalent series resistance directly as series values.

(3) *Equivalent shunt capacitance.*

Increase frequency well above pass-band. Interchange C and R. Use as simple capacitor bridge.

Measurements can be made with a polarizing direct current through the windings if a source of d.c. is connected to A and D. Of course, the d.c. path through E should then be blocked with a capacitor and the d.c. supply must be fed through a choke to present a high impedance to E.

Results Obtained

Figure 5 shows two curves taken in the manner described. Curve 1 is the measured distortion characteristic of a typical Partridge High Fidelity output transformer (type PPO/2) employing a high grade silicon-iron core. Curve 2 is the same characteristic taken for the same transformer but with a Radiometal core substituted for the silicon iron. It will be noticed that really low percentages are obtained by the use of the more expensive nickel-iron core material which also greatly increases the band width obtainable with any given degree of primary-secondary intersectioning. It will be noted, however, that the power handling capacity of the transformer is not increased by the use of the nickel iron core. In both cases the percentage distortion increases rapidly for power levels above 10 watts. This should be expected as both materials saturate at about the same flux density.

Harmonic Distortion and Intermodulation

It is now widely recognized that intermodulation distortion is more important than harmonic distortion as a cause of poor fidelity in audio reproducing systems. In conclusion therefore it may be as well to justify the attention paid to the measurement of harmonic distortion.

It is well-known that harmonic distortion arises from the non-linearity of the magnetic properties of core materials in common use. The production of harmonics is, in itself, not very objectionable since almost all transmitted sounds are rich in harmonics to various degrees; for example, the note of a violin is characterized by the presence of certain harmonic frequencies in proportions which vary for different instruments and for the same instrument played differently. Hence, the propor-

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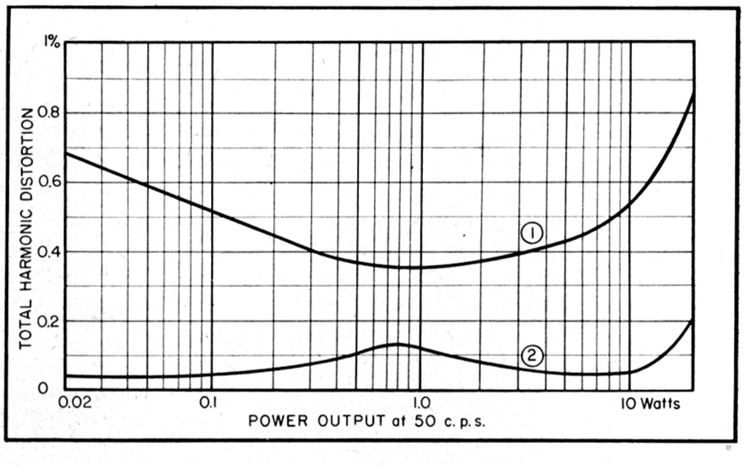
Fig. 5. Typical distortion curves taken on two transformers with identical windings and different core materials. (1) silicon-iron core, and (2) Radiometal core.



HARMONIC DISTORTION

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tion of harmonics can be varied within wide limits without destroying the realism of the reproduced sound.

However, the effect upon a compound tone of harmonic distortion in the reproducing system is more objectionable since it is inevitably accompanied by the production of other tones which are not musically related to the fundamental.

These tones are known variously as "intermodulation tones," "combination tones" and "sum-and-difference tones." Possibly the worst result of their presence (in addition to their unmusical character) is the effect on the clarity of reproduction. Individual tones lose distinction and become merged, one with another, due to the wide spectrum of sum-and-difference tones.

It would seem that the best way of specifying the distortion introduced by a circuit element would be to measure the intermodulation distortion. However, it is most desirable to express the distortion by a single number which will offer a direct comparison between equipments; this cannot be done for intermodulation distortion without making some very arbitrary assumptions. To measure intermodulation distortion the procedure is:

(1) Feed in *two* test tones simultaneously, each of known amplitude and frequency (four numbers).

(2) Measure the r.m.s. value of combination tones and harmonics appearing at the output. This is usually expressed as a percentage of the r.m.s. input voltage (one number).

Five numbers are therefore required for the complete specification of intermodulation distortion.

Total harmonic distortion, on the other hand, is completely expressed by three numbers; frequency and amplitude of test tone and r.m.s. value of harmonics appearing in output. The obvious choice of frequency is that of the lower limit of the pass-band, and the amplitude can be that corresponding to maximum rated output; these figures must be stated anyway, and only one more figure is needed to indicate the harmonic distortion.

Since the two types of distortion are so closely related, the magnitude of the harmonic distortion is some indication of the extent of the more objectionable intermodulation, and is more easily measured. Neither figure, of course, is important in itself; the measure of electrical distortion must be correlated to the degree of aural "nuisance value" to which it corresponds.