Using the Reactance Chart for Filter Design Problems

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Calculations of components for various filter and equalizer circuits are simplified by graphical means.

THE VALUE OF the reactance chart such as that shown in *Fig.* 1 is becoming more and more widely recognized as time goes on. Although it was originally indicated by Slonczewsky that the chart could be used for other purposes than reactance and resonant-frequency calculations, it is still largely used for that purpose. The other short-cuts offered by the chart have been largely neglected.

It has been known for some time that these charts could be used for certain filter calculations. It is the purpose of this article to show why this is true and how it may be done for several popular types of filters. Although the material may not be new to many readers, it is felt that there are others to whom the information may be of value.

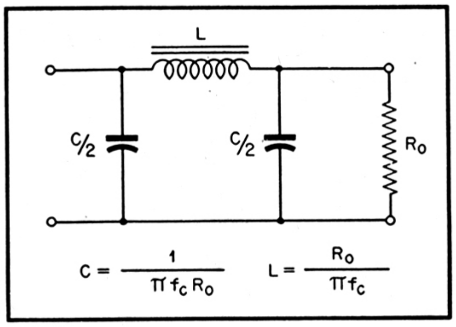


Fig. 2. Low-pass filter prototype, or constant-*k* section. *Ro* is the terminating resistance; *fc* is cut-off frequency.

The chart is designed to solve equations of three parameters in which 2π enters as a factor. Specifically it solves such equations as

$$X_L = 2\pi fL \quad (1)$$

$$X_C = \frac{1}{2\pi fC} \quad (2)$$

$$f = \frac{1}{2\pi\sqrt{LC}} \quad (3)$$

Solving for L and C in equations (*1*) and (*2*) gives

$$L = \frac{X_L}{2\pi f} \quad (4)$$

$$C = \frac{1}{2\pi f X_C} \quad (5)$$

The chart may be used to solve these or other three-parameter equations having 2π as a factor if the equation to be solved can be put into one of these or an equivalent form.

Inductive and capacitive reactance have the dimensions of resistance. Therefore, without changing the form of the equations of the ordinate of the chart equations, (*1*) and (*2*) may be written

$$R_1 = 2\pi fL \quad (6)$$

$$R_2 = \frac{1}{2\pi fC} \quad (7)$$

Solving equations (*6*) and (*7*) for L and C respectively gives

$$L = \frac{R_1}{2\pi f} = \frac{1}{2} \bullet \frac{R_1}{\pi f} \quad (8)$$

$$C = \frac{1}{2\pi f R_2} = \frac{1}{2} \bullet \frac{1}{\pi f R_2} \quad (9)$$

Equations (*8*) and (*9*) are seen to be very similar to the equations for the low-pass filter prototype shown in *Fig.* 2. The equations indicate, however, that the values given by the reactance chart for L and C will be equal to one half of the value calculated from the prototype equations if R_o is substituted for R_1 and R_2 and f_c is substituted for f.

For example, if it is desired to design a 10,000-ohm filter section with a cut-off frequency f_c of 1,000 cps, substituting these values in the equations of *Fig.* 2 give

$$L = \frac{10{,}000}{\pi \times 1000} = 3.18H, \text{ and}$$

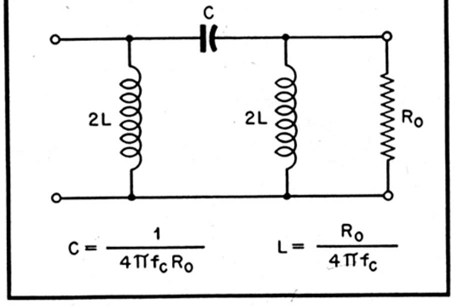


Fig. 3. High-pass filter prototype, or constant-*k* section.

$$C = \frac{1}{\pi \times 1000 \times 10{,}000} = 0.0318\ \mu f$$

Using the reactance chart and entering the chart at R_o=10,000Ω at the intersection with the 1000 cps vertical, it

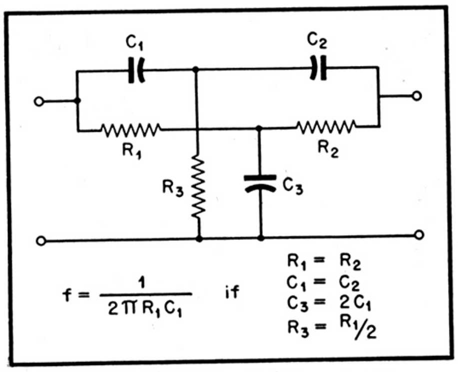
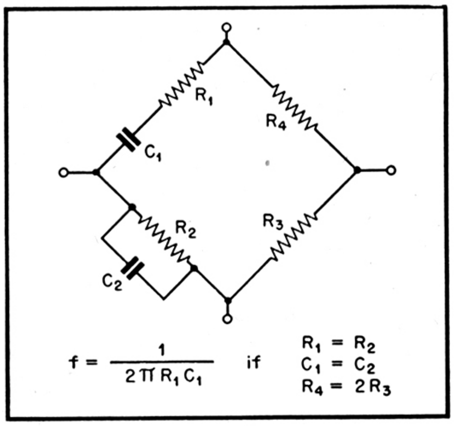


Fig. 4 (above). Parallel-T rejection or null network configuration.

Fig. 5. (below). Wien bridge circuit as employed in *RC* oscillators.



is found that L is given as 1.59 Henrys and C is 0.0159 μf. These values are exactly half of the values found from the equations.

In a similar manner, if f_c is substituted for f and R_o for X_L and X_c in equations (*4*) and (*5*), equations (*10*) and (*11*) result.

$$L = \frac{R_o}{2\pi f_c} \quad (10)$$

$$C = \frac{1}{2\pi f_c R_o} \quad (11)$$

These equations are seen to be simi-

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lar to the high-pass prototype equations shown in *Fig.* 3. Dividing both sides of equations (*10*) and (*11*) by two yields

$$\frac{L}{2} = \frac{R_o}{4\pi f_c}$$

$$\frac{C}{2} = \frac{1}{4\pi f_c R_o}$$

Solving these equations for L and C gives

$$L = 2 \times \frac{R_o}{4\pi f_c}$$

$$C = 2 \times \frac{1}{4\pi f_c R_o}$$

From these equations it is seen that the values of L and C obtained from the reactance chart will be exactly twice the value obtained from the high-pass filter prototype equations. That this is true may be verified by solving the equations for the filters and comparing the results with the values obtained from the chart.

If equation (*2*) is solved for f and

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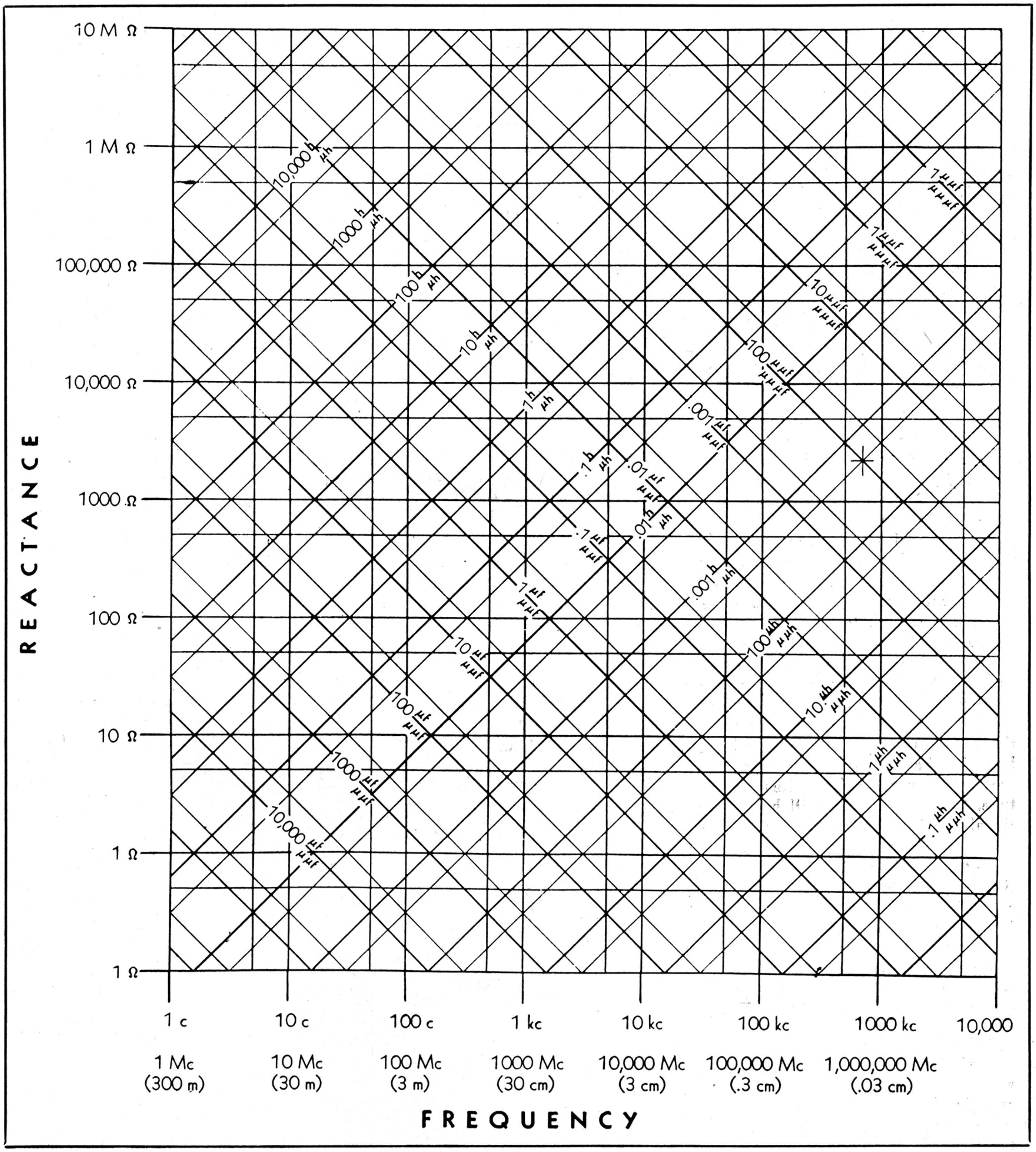

Fig. 1. The reactance chart, well known for its many applications in circuit calculations.

(Courtesy General Radio Company)

the cosine of these phase shift angles. In some cases this may cause instability of the amplifier, particularly where large amounts of feedback are used. Experience has shown that the phase shift begins to be measureable at values 1/7th to 1/10th of the frequency at which the 2 db point shows up on a gain-frequency characteristic. Therefore, the designed bandwidth should be from 7 to 10 times the highest frequency for which it is desired to have distortion less than 1 per cent. The figure shows that the phase shift through the amplifier is substantially zero from 30 to 30,000 cps.

The circuit here described in part appears to open new fields of use or improvement in present fields permiting operation very near the theoretical maximum efficiency and yet provides a high degree of linearity with high stability for either impulse or steady state signals.

USING THE REACTANCE CHART

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R substituted for X_c the equation becomes

$$f = \frac{1}{2\pi RC}$$

This equation will be recognized as the equation for the rejection frequency of the parallel-T filter, such as is shown in *Fig.* 4 providing $C_1 = C_2$ and $R_1 = R_2$ with R_3 and C_3 properly proportioned. This filter is electrically equivalent to the Wein bridge shown in *Fig.* 5; consequently, the chart may also be used to determine the null frequency of such a bridge, or the frequency of an oscillator employing the Wein bridge for the frequency-determining element.

Thus, it has been shown that the conventional reactance chart may be used without modification for determining component values for the low-pass filter prototype if the values of L and C read from the chart are taken for $L/2$ and $C/2$. In the case of the high-pass filter the values indicated by the chart are twice the values given by the filter equations. The value of the series elements of a parallel-T rejection filter or Wein bridge, such as is used frequently in oscillators, are indicated directly on the chart.

Although graphical methods are not recommended for highly accurate work, the results obtained by using the chart are sufficiently accurate for many engineering applications.