Learning to love the deciBel

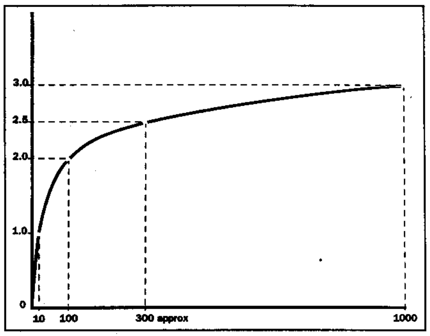
John Watkinson begins a new tutorial series by looking at why the deciBel tends to be universally misunderstood

TELEPHONY was the first extensive application of audio engineering. As a result, as soon as long telephone wires were erected between towns it became clear that they had quite different characteristics to the short wires in a lab. This is because they were acting as transmission lines. It was discovered that the power available from the other end fell logarithmically with distance. Before a complete understanding was available, comparative measurements were made using a 'mile of standard cable' or MSC. This consisted of a mile of 'twenty pound' copper wire which had a loop resistance of around 88Ω (Ohms) and a capacitance of about 54nF (nanoFarads) per mile. Tests showed that such a cable would output just under 80% of the input power when measured at a mid-band (for speech) frequency of 800cps (cycles per second, now Hertz). Clearly if several such MSCs are connected in tandem, the output power will be 0.8 x 0.8 x 0.8... of the input, hence the logarithmic characteristic.

Fig.1 shows the principle of the logarithm. To give an example, if it is clear that 10^2 is 100 and 10^3 is 1000, then there must be a power between 2 and 3 to which 10 can be raised to give any value between 100 and 1000. That power is the logarithm to base 10 of the value. For example, $\log_{10} 300 = 2.5$ approx. Note that 10 to the power of zero is 1.

Logarithms were developed by mathematicians before the availability of calculators or computers to ease calculations such as multiplication, squaring, division and extracting roots. The advantage is that armed with a set of log tables, multiplication can be performed by adding, division by subtracting. **Fig.1** shows some examples. It will be clear that squaring a number is performed by adding two identical logs and the same result will be obtained by multiplying the log by 2.

The slide rule shown in **Fig.1** is an early solar-powered solid-state calculator which consists of two logarithmically engraved scales in which the length along the scale is proportional to the log of the engraved number. By sliding the moving scale two lengths can easily be added or subtracted and as a result multiplication and division is readily obtained.



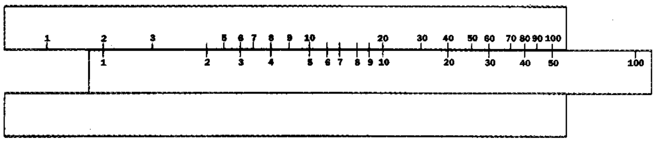


Fig.1:

Log principles and a slide rule

By 1923 the physics of the telephone transmission mechanism were understood and Bell Telephone engineers proposed a 'transmission unit' defined as the output power being one tenth the input. Following international consultation the unit was named the Bel in honour of Alexander Graham Bell [**1**]. Human hearing (and indeed other senses) also has a logarithmic response with respect to sound pressure level (SPL). In order to relate to the subjective response audio signal level measurements have also to be logarithmic and so the deciBel was adopted for audio level display and signal measurement.

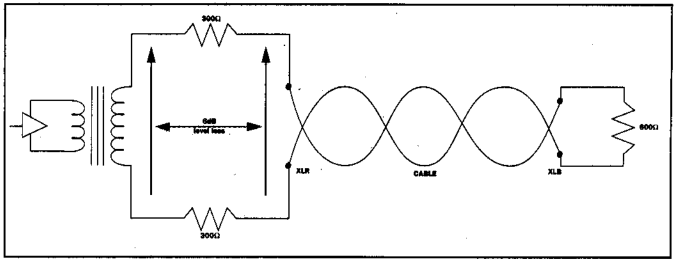


Fig.2a: Impedance matching is undesirable

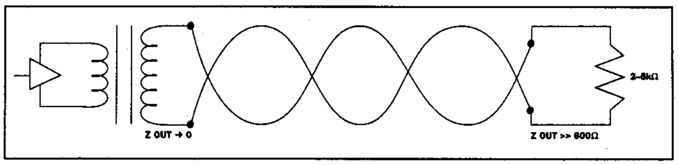


Fig.2b: Proaudio – sources have lowest output impedance

The Bel was (and is) defined as shown in the Panel as the logarithm to base 10 of the ratio of input power to output power. It is useful to remember that one Bel is a 10:1 power ratio. It must be stressed that the dB is dimensionless because it is a ratio. The Bel is fine for long telephone lines where heavy attenuation is experienced, but for other purposes it is a large unit. As with large units like the Farad it was diminished by a prefix, the most common being 'deci', meaning one tenth, hence the deciBel, abbreviated dB and pronounced dee-bee. One of the factors which encouraged the adoption of the dB was that the loss in an MSC was very nearly 1dB. The solution was to raise the test frequency to 886cps so that the loss increased slightly to make the MSC have exactly 1dB of loss.

Thus the loss in a cable in dB could be quoted by measuring the input power and the output power and multiplying the log of the ratio by ten. When the two ends of the cable are miles apart it makes sense to have a standard power so one engineer can insert a standard amount of power and his mate can measure it in the next town.

One milliWatt (1mW) was chosen as a practical standard. To show that the measurement is not a ratio of two measurements but the ratio between a single measurement and a fixed reference, the unit has to be qualified. Thus where the reference is 1mW, the units will be dB(m). In radio engineering, the dB(W) will be found which is power relative to one Watt.

A device such as an amplifier can have a fixed power gain which is independent of signal level and this can be measured in pure dB. However, when measuring the power of a single signal, it must be appreciated that the deciBel is a ratio and to quote the number of dBs without stating the reference is about as senseless as describing the height of a mountain as 2,000 without specifying whether this is feet or metres.

In long cables the distributed series inductance and the parallel capacitance interact to give the line a characteristic impedance. In telephones this turned out to be about 600Ω. In transmission lines the best power delivery occurs when the source and the load impedance are the same; this is the process of matching. Thus a standard condition against which signals could be compared was the dissipation of one milliwatt in 600Ω. The side bar shows that the dissipation of 1mW in 600Ω will be due to an applied voltage of 0.775V rms.

The side bar also shows that as the power is proportional to the square of the voltage, the power ratio will be obtained by squaring the voltage ratio. As squaring in logs is performed by doubling, the squared term of the voltages can be replaced by multiplying the log by a factor of two. To give a result in deciBels, the log of the voltage ratio now has to be multiplied by 20.

While 600Ω matched impedance working is essential for the long distances encountered with telephones, it is quite inappropriate for audio wiring in a studio. The wavelength of audio in wires at 20kHz is 15 kilometres. Most studios are built on a smaller scale than this and clearly analogue audio cables are not transmission lines and they do not have a characteristic impedance. Readers should treat anyone who attempts to sell exotic analogue audio cables by stressing their transmission line characteristics as free entertainment.

In professional audio systems impedance matching is unnecessary and undesireable. **Fig.2a** shows that when impedance matching is required the output impedance of a signal source must be artificially raised so that a potential divider is formed with the load.

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< page 99 The actual drive voltage must be twice that needed on the cable as the potential divider effect wastes 6dB of signal level and requires unnecessarily high power supply rail voltages in equipment. A further problem is that cable capacitance can cause an undesireable HF roll-off in conjunction with the high source impedance.

In modern professional audio equipment, shown in **Fig.2b**, the source has the lowest output impedance practicable. This means that any ambient interference is attempting to drive what amounts to a short circuit and can only develop very small voltages. Furthermore, shunt capacitance in the cable has very little effect. The destination has a somewhat higher impedance (generally a few kΩ) to avoid excessive currents flowing and to allow several loads to be placed across one driver.

In the absence of a fixed impedance it is now meaningless to consider power. Consequently, only signal voltages are measured. The reference remains at the historically derived 0.775 volts, but power and impedance are irrelevant. Voltages measured in this way are expressed in dB(u); the commonest unit of level in modern audio systems. Most installations boost the signals on interface cables by 4dB. As the gain of receiving devices is reduced by 4dB, the result is a useful noise advantage without risking distortion due to the drivers having to produce high voltages.

In order to make the difference between dB(m) and dB(u) clear, consider the lossless matching transformer shown in **Fig.3**. The turns ratio is 2:1 therefore the impedance matching ratio is 4:1. As there is no loss in the transformer, the power in is the same as the power out so that the transformer shows a gain of 0dB(m). However, the turns ratio of 2:1 provides a voltage gain of 6dB(u). The doubled output voltage will develop the same power in to the quadrupled load impedance.

In the digital domain there is no audio impedance, voltage or power; only numbers. Consequently, the dB(m) and the dB(u) are of no use at all. Instead digital level is measured as shown in the side bar by the ratio of the signal amplitude to the largest possible sinusoidal amplitude which is defined as 0dB(Fs) or full scale.

In acoustic measurements, the sound pressure level (SPL) is measured in deciBels relative to a reference pressure of 2 x 10^{-5} Pascals (Pa) rms. In order to make the reference clear the units are dB(SPL). In measurements that are intended to convey an impression of subjective loudness, a weighting filter is used prior to the level measurement that approximates the frequency response of human hearing which is most sensitive in the mid range. The most common standard frequency response is the so-called A-weighting filter, hence the term dB(A) used when a weighted level is being measured. At high or low frequencies, a lower reading will be obtained in dB(A) than in dB(SPL). ■

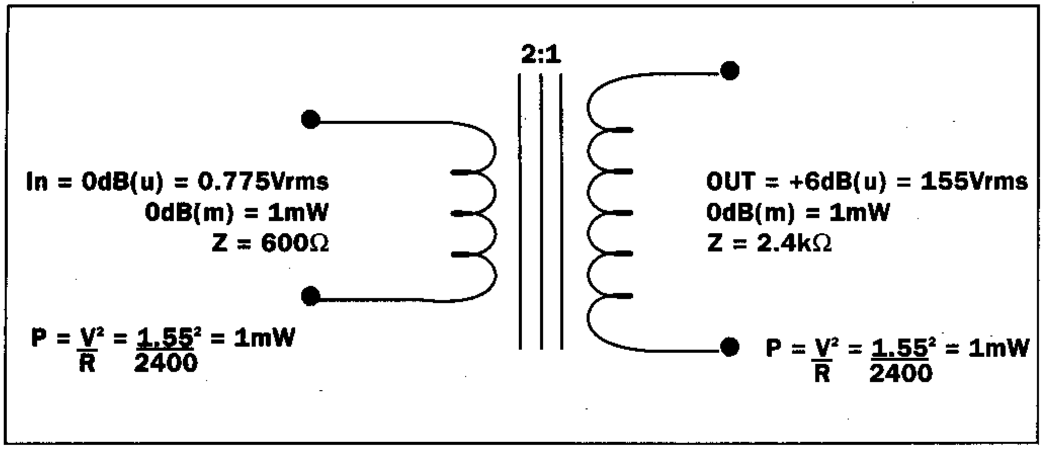


Fig.3: Lossless matching transformer

References

1. Martin, WH, 'DeciBel–the name for the transmission unit.' *Bell System Technical Journal*, Jan 1929.

$$\text{Power Gain in BELS} = \log_{10} \frac{\text{P in}}{\text{P out}}$$

$$\text{Power Gain in dB} = 10 \log_{10} \frac{\text{P in}}{\text{P out}}$$

Assuming 1mW power reference

$$\text{Power in dB(m)} = 10 \log_{10} \frac{\text{Power}}{1/1000} = 10 \log_{10} (\text{Power} \times 1000)$$

$$\text{As Power } P = V^2/R \text{ and } V = \sqrt{RP}$$

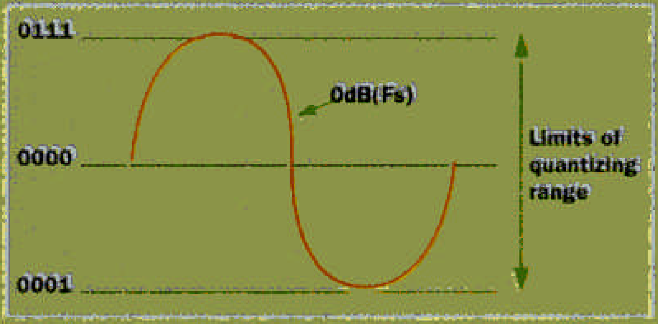
$$\text{Power Gain (dB)} = 10 \log_{10} \frac{\text{V in}^2/R}{\text{V out}^2/R}$$

$$= 10 \log_{10} \frac{\text{V in}^2}{\text{V out}^2} = 20 \log_{10} \frac{\text{V in}}{\text{V out}}$$

$$\text{If } P = 1\text{mW and } R = 600\Omega$$

$$V = \sqrt{RP} = \sqrt{600 \times 0.001} = \sqrt{0.6} = 0.77459..\ \text{Vrms}$$

$$\text{Level in dB(u)} = 20 \log^{10} \frac{\text{Voltage rms}}{0.77459}$$



In digital systems there is no audio voltage, power or impedance. Level is measured in dB (Fs) shown here as largest sine wave which fits quantizing range

< page 99 The actual drive voltage must be

In order to make the difference between